The manipulation of closing prices ☆

Pierre Hillion, Matti Suominen*

INSEAD, Department of Finance, Boulevard de Constance, 77305 Fontainebleau, France

Abstract

Before the introduction of a call auction at the close, the last minute of trading at the Paris Bourse was the most active of the whole day. Even though the bid–ask spread increased substantially, the probability of large and aggressive orders increased, as did price volatility. In addition, both the one-minute returns and the proportion of partially hidden orders increased. In this paper, we develop an agency-based model of closing price manipulation, which can account for these phenomena. In addition, we discuss the optimal closing price mechanism under manipulation.
© 2004 Elsevier B.V. All rights reserved.

JEL classification: G10; G24

Keywords: Manipulation; Closing price; Call auction

1. Introduction

In this paper we develop an agency-based model of closing price manipulation. In addition, we study the determination of the closing price in the presence of

*Corresponding author. Tel.: +33-1-6072-4000.

E-mail addresses: pierre.hillion@insead.edu (P. Hillion), matti.suominen@insead.edu (M. Suominen).

manipulation, under different closing price mechanisms. The determination of the closing price is important, as this is the price that is most closely followed by investors at large. In addition, it is the price most commonly used by academic researchers and a reference price against which fund managers are often evaluated.

Our research is motivated by recent changes in the closing price mechanisms in many stock exchanges around the world, triggered by concerns from market participants of unfair price setting. For example, in the Paris Bourse, prior to the introduction of a call auction at close, there was a striking jump in price volatility during the last minute of trading. This was accompanied by equally striking jumps in the order submission rate and the trading volume, despite an increase in the bid–ask spread.[1]

We believe that these observations are related, in part, to the manipulation of closing prices. In this paper, we analyze one particular form of closing price manipulation which, according to practitioners, is common: manipulation by brokers. We present a model in which a broker manipulates the closing price to alter the customer's inference of his execution quality.[2]

To begin with, let us briefly discuss the role of brokers in market liquidity creation in stock markets. Grossman and Miller (1988) argue that, while market makers, or dealers, are able to provide market liquidity in the case of small changes in the supply or demand for a stock, the large blocks are typically traded in the upstairs market by traders of the leading investment banks. In these markets, traders basically "shop the block" among their customers, and when a suitable counterparty or counterparties are found, the trade is reported to the stock exchange, or, in the case of an electronic market, both the sell and buy orders are entered into the electronic trading system. For large blocks, this process, which allows time for more market participants to come to the market, produces a smaller price impact from the trade as compared to selling the block directly on the organized stock exchange.[3]

In our model, a broker's execution quality depends on his ability to search out additional traders in the market, to accommodate his customer's order. The customer does not know her broker's ability, but tries to estimate this from the prices that she observes: her execution price and the closing price. The broker, in turn, tries

---

[1] Other findings were that both the one-minute returns and the proportion of hidden orders, i.e., orders where the magnitude of the order is not revealed to the market before execution, were at their highest in the last minute of trading. In addition, the frequency of large and aggressive orders, i.e., orders whose limit price and magnitude are such that a trade and a change in the highest bid or lowest ask occurs, was at its highest during the last 10 seconds of trading.

[2] Independently, in an empirical paper, Felixson and Pelli (1999) also argued that brokers, or their customers, manipulate the closing prices to show higher trading profits. It has also been argue that there is manipulation near the close on days when derivatives contracts expire. See e.g., Kumar and Seppi (1992). The above observations on the Paris Bourse are made even when the expiration dates of futures and options are excluded.

[3] See, however, also Madhavan and Cheng (1997) and Seppi (1990), who suggest other roles for upstairs markets.

to influence the customer's learning about his ability, by exerting effort when executing the order and by manipulating the closing price.[4]

In the case of a customer's sell order, for instance, a low execution price, as compared with the closing price, is an indication of low broker ability. Because the broker's future commission depends on the customer's estimate of his ability, the broker, in this case, manipulates the closing price downwards by selling shares at the close. Manipulation affects prices as market participants are risk averse. In our model, the customer understands that manipulation takes place and, despite the manipulation, is able to make the right inference on the broker's ability. Nevertheless, in equilibrium, manipulation occurs.

This leads us to study the effect of the closing price mechanism on the closing price. In our model, the introduction of a call market at the end of the trading day, as was done at the Paris Bourse, reduces manipulation and brings the closing price nearer to the fair value of the asset at close. Our results suggest, however, that optimally, the trading interruption prior to the call market should depend, among other things, on the liquidity of the stock (which it does not in Paris).

The contribution of this paper is two fold. The first contribution is theoretical. The paper develops an agency-based model of closing price manipulation. Until now, the theoretical literature on market manipulation has ignored manipulation for agency reasons as a potential reason for manipulation.[5] The second contribution is empirical. The paper complements the evidence uncovered about the close in the US stock markets, documented by Harris (1989), McInish and Wood (1990) and Cushing and Madhavan (2000), by looking at a different geographical market, with a different market structure and a tighter time scale. In addition, we provide evidence which suggests that these phenomena are related to market manipulation. Since our model is consistent with many of the observed patterns in volatility, returns and volume during the last minute of trading, it also contributes to the theoretical literature on intraday price and volume dynamics, such as studies by Admati and Pfleiderer (1988) and Hong and Wang (2000).

The paper is organized as follows: Section 2 presents the empirical evidence from the Paris Bourse. Section 3 develops a model of market manipulation and discusses both its predictions and key assumptions. Section 4 studies the optimal closing price mechanism and presents evidence from Paris and Madrid on the effects of changes in the closing price mechanisms. Section 5 concludes. All proofs and a description of closing procedures in selected exchanges are provided in the appendix.

---

[4] Investor service companies that evaluate brokers' execution quality also evaluate brokers by comparing their execution prices to the closing prices (or to the average of the Open, Close, High and Low). See McSherry and Sofianos (1998).

[5] See, e.g., Cherian and Jarrow (1995) for a survey of the literature.

2. Anomalies at the close in the Paris Bourse

In this section, we document and discuss some of the empirically observed intraday patterns at the Paris Bourse before the introduction of a call auction at close.

Until June 1998, there was continuous trading at the Paris Bourse after the opening call auction, and the closing price was determined simply as the last transaction price of a trading day. In June 1998, a second call auction was introduced to determine the closing price. To investigate the reason for this, we collected a sample of intraday stock prices prior to the introduction of the call auction. The sample includes the 40 stocks that compose the CAC40 stock index (the most liquid stocks). For each stock, it contains every single order submitted to the Paris Bourse for the period between January 3, 1995 and April 26, 1995.

Fig. 1 shows the pattern of intraday volatility within our sample period. As it shows, there is a striking increase in volatility of stock returns, measured from the midpoints of the bid–ask spread, at the very last minute of trading. This accords with previous studies that have found a U-shaped pattern for intraday volatility, such as Admati and Pfleiderer (1988), but shows that, at least for our sample, the rise in volatility near the close occurs mainly in the very last minute of trading.

Whether we look at the order submission rate, the number of transactions, or the volume of trade, the picture is similar. For instance, over 2.5% of all transactions that take place during the continuous market occur in the last minute of trading. This is ten times the unconditional average and over three times the intraday high excluding the last two minutes of trading. Similar to volatility, the finding that intraday trading volume is U-shaped is not new. However, it has not been noted that the increase in volatility and trading volume near the close occurs mainly in the last minute of trading.

Brock and Kleidon (1992) and Hong and Wang (2000) develop models in which the overnight closure of markets gives rise to an increase in volatility and trading
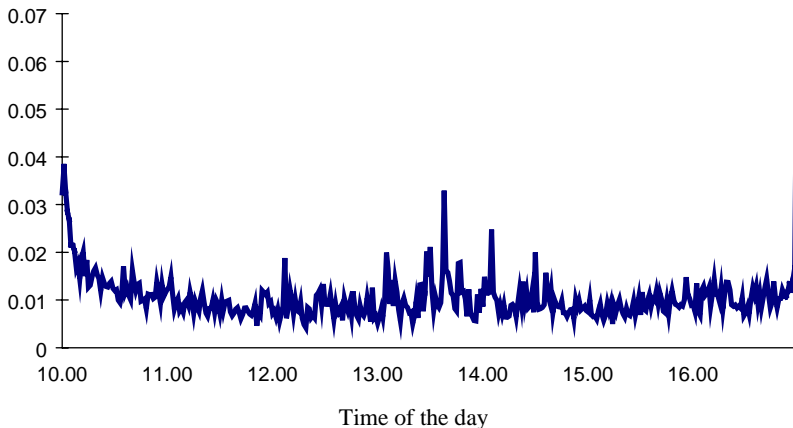

Fig. 1. One-minute variance of the CAC40 stocks (100 × return squared), January 3, 1995–April 26, 1995.
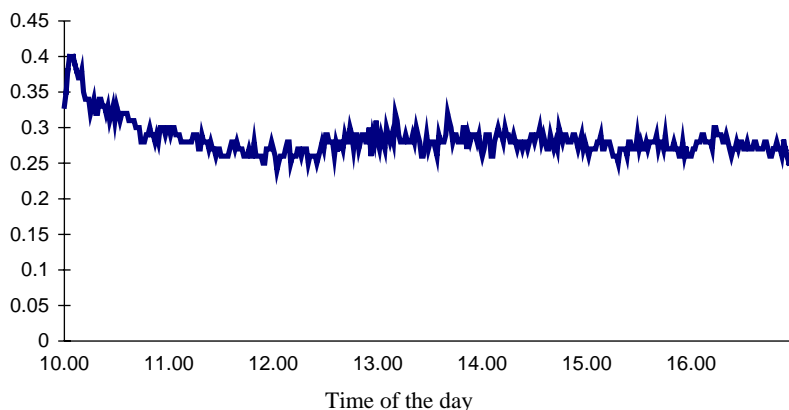
Fig. 2. Relative bid–ask spread of the CAC40 stocks (%), January 3, 1995–April 26, 1995.

volume near the close.[6] They argue that traders reduce their positions before the market closure to avoid taking excessive risks overnight, when they cannot alter their positions. Admati and Pfleiderer (1988) offer another theory to explain the clustering of trades near the market closure. They argue that liquidity traders cluster their trades to reduce the adverse selection problem they face when trading against informed traders. This clustering, they argue, brings additional informed traders to the market, which leads to increased price volatility.

We have two other empirical results from the Paris Bourse that cast doubt on these two explanations for the last-minute rise in volatility and trading volume. First, as Fig. 2 shows, there is a significant increase in the bid–ask spread during the last minute of trading. This suggests that the last minute of trading is hardly the best period for the liquidity traders to trade. Traders who want to reduce their overnight holdings would be better off reducing them two or three minutes before the market closure, rather than during the last minute of trading.

A second result which suggests that something different occurs near the close is depicted in Fig. 3. This figure shows a rise in the percentage of hidden orders during the last minutes of trading.[7] It seems that at close many traders do not want to advertise their imbalances to attract counter demand but, for some reason, want to make the market look as thin as possible.

3. The model

In this section we describe a formal model of equity markets, with an explicit role for a broker to reduce the price impact of his customer's trades. We characterize an

---

[6] See Foster and Viswanathan (1990) for other effects of market closure on volume and volatility.

[7] These are orders whose magnitude is not revealed to the market before execution. The increase in hidden orders explains a large part, but not all, of the increase in the total number of orders just before the market closure.
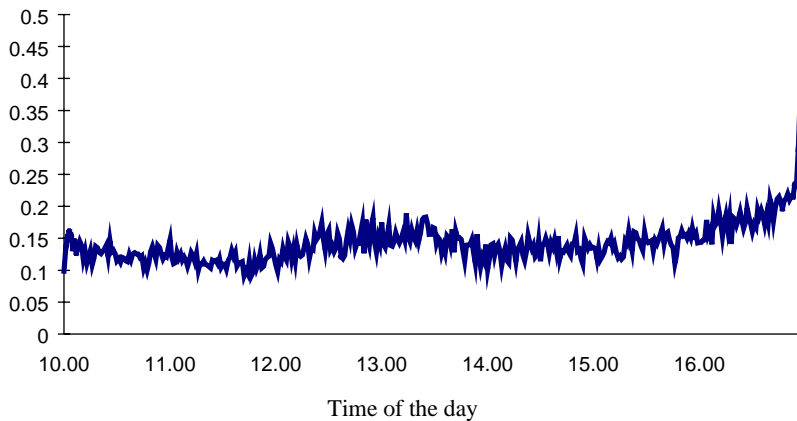
Fig. 3. Proportion of hidden orders for CAC40 stocks, January 3, 1995–April 26, 1995.

equilibrium in which the broker exerts an effort when executing his customer's order, and manipulates the closing price in order to influence his customer's perception of his performance. We then discuss the empirical predictions of the model and, finally, the robustness of the model with respect to its assumptions.

The motive and the direction of manipulation can be understood from the following example: Suppose the closing price yesterday was $100, and today the customer sells a block of shares at $90 per share. At the end of the day, the customer tries to estimate the price impact of her trade and the broker's ability from the prices that she observes, i.e., yesterday's close, her execution price, and today's closing price. Now, if today's closing price is $90, the customer is more likely to think that the execution quality was good than if the closing price is $100. In the first case, the low execution price is more likely to be due to unfavorable shocks to the value of the asset before the execution of the order, than in the latter case. Because of this, to alter the inference process in his own favor, the broker manipulates today's closing price downwards by selling shares at the end of the trading day.

3.1. The set-up

We now describe the basic assumptions of the model: assets, agents, their information sets, objective functions, timing of the game, and the concept of equilibrium.

Assets and security market: There are two assets: a single risky asset, which is in zero net supply, and a safe asset, with an infinitely elastic supply. The return on the safe asset is normalized to zero. The assets are traded in a security market, organized by an exchange, during two days $d \in \{0, 1\}$, after which they are liquidated. Within the two days, there are $2T$ trading periods, period T being the last period of day zero. At the beginning of each period $t \in \{1, ..., 2T\}$, there is an innovation $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ to the value of the risky asset. Its liquidation value, at the end of period

$2T$, v, is the sum of a fixed value, $\bar{v}$, and the periodic innovations:

$$v = \bar{v} + \sum_{t=1}^{2T} \varepsilon_t. \tag{3.1}$$

Agents, who are present in the security market in period t, may post limit orders, which indicate their demand for the risky asset, conditional on the period t market price, P_t. The market price, P_t, is selected by the exchange as follows: If the market clears at several different prices, P_t is the highest of them. If there is no market clearing price, $P_t = \bar{v} + \sum_{s=1}^{t} \varepsilon_s$. In this case, agents' oversupply (overdemand) is proportionally rationed. It turns out that there is a unique market clearing price in the equilibrium that we describe.

Agents: There are four groups of agents. The first contains one customer. This customer arrives only in periods 1 and $T+1$ and at no other time. There are no other customers in this model. The second and third agents are an active broker A, and a discount broker D. Finally, the fourth group holds several limit order traders.[8] All agents live for two days and have a constant absolute risk aversion. Their utility for wealth W at the end of period $2T$ is

$$U = -\exp(-aW), \tag{3.2}$$

where a is the parameter of risk aversion. There is no discounting. All agents are initially endowed with zero units of the two assets. There is limited security market participation so that only some of the agents are present in the market at any point in time.

Customer: The customer can only trade via a broker. At the beginning of both days $d \in \{0, 1\}$, i.e., in periods 1 and $T+1$, she receives a random daily endowment of $x^d \in \{\bar{x}^d, -\bar{x}^d\}$ units of the risky asset, which she must immediately sell to (or buy from) traders using a broker $b^d \in \{A, D\}$. The broker charges a commission $C_{b^d}^d$ for this service. Here $\bar{x}^d$ is a positive constant, known by all agents, and the probability that $x^d = \bar{x}^d > 0$ is one half. The trading needs of the customer are not modelled.

The periodic shocks, ε_t, and security prices, P_t, are observable to all traders in the market, but not to the customer. The customer observes only the closing prices and the execution prices of her trades. Denote by Ψ_t her (beginning of) period t information set. The information set Ψ_{dT+1} includes her daily endowment, x^d, and, if $d = 1$, her day zero broker, b^0, day zero transaction price, P_1, brokers' day zero commissions, $\{C_b^0\}_{b=A,D}$, and the closing price P_T. In period $dT+1$, after observing the brokers' day d commissions, her objective is to maximize her expected utility (3.2), by selecting her day d broker $b^d \in \{A, D\}$. That is, she maximizes

$$\max_{b^d \in \{A,D\}} \mathrm{E}\left(-\exp\left(-a \sum_d \left[x_d P_{dT+1} - C_{b^d}^d\right]\right) \middle| \Psi_{dT+1}, C_A^d, C_D^d\right). \tag{3.3}$$

Traders: In each period t, only a limited number of traders, m_t, are present in the market. First, in each period, m new traders arrive in the market and then exit.

---

[8] Broker D is simply a mechanism to discipline the pricing power of broker A. Instead of broker D, as discussed in Section 3.5, we could have assumed the existence of two competing active brokers.

Second, in each period, broker A may search out additional traders in the market. In this case, $m_t > m$. The traders who are present in the market post limit orders to trade the risky asset on their own account. In effect, they perform the traditional function of a market maker, by providing liquidity into the market.

A period t trader's objective is to maximize his expected utility (3.2) by choosing his demand schedule for the risky asset, $z_t(P : P = P_t)$, conditional on his period t information set, Ω_t. Here Ω_t includes Ψ_t, the current period's shock, ε_t, and all past shocks, ε_s, and prices, P_s, where $s < t$. We assume that the traders behave competitively (we can think that the number m is large) and ignore their own actions' impact on prices. Letting E_t denote the expectations operator conditioned on Ω_t, a period t trader's maximization problem is

$$\max_{z_t(P)} E_t - \exp(-a z_t(P_t)(v - P_t)). \tag{3.4}$$

Brokers: The brokers act as intermediaries between the customer and the traders. At the beginning of the periods $dT + 1$, they compete for the customer's day d order by setting their day d commissions, $\{C_b^d\}_{b=A,D}$. One of them is then selected by the customer to execute her order. Both brokers' beginning of the period t information set is the same as that for the traders, i.e., Ω_t. Their cost of period t effort, e_t, is $e_t^2/2$.

Broker D (discount broker): The discount broker D cannot influence the number of traders in the market, nor trade on his own account. If selected by the customer, he simply sells (buys) the customer's block to (from) the m traders in the market, at the market price. This requires an effort ϱ from him.

Denoting by $\pi_d(\{C_b^d\}_{b=A,D})$ the probability that $b^d = D$, broker D's objective at time $t = dT + 1$, is to set his commission C_D^d to maximize his expected utility, i.e.,

$$\max_{C_D^d} E_{dT+1} - \exp\left(-a \sum_d \pi_d \left(C_D^d - \frac{\varrho^2}{2}\right)\right). \tag{3.5}$$

Broker A (active broker): In period $dT + 1$, broker A may also be selected to execute the customer's day d order. When executing her order, broker A may, as broker D does, simply trade with the m traders who are already in the market, by exerting effort ϱ. Alternatively, he may search out $m_t - m$ additional traders in the market, and trade with a larger group of $m_t > m$ traders.

The number of additional traders that broker A brings to the market in period t depends on his ability and action. If broker A has an ability q^{-1}, he can increase m_t from m to $m_q(e_t) = (q - e_t)^{-1}$, and trade with m_q traders, by exerting effort $e_t \geqslant \varrho$ in period t.[9] We assume that it is common knowledge among agents that $q \sim N(\bar{q}, \theta^2)$, independent of v, and $0 < \bar{q} - \varrho < 1/m$. This implies that neither the customer nor broker A himself perfectly knows broker A's ability. We may imagine, for instance, that broker A has already communicated to his customer the part of his ability that

---

[9] Note that broker A can increase the number of traders to $m_q(\varrho) > m$, and trade with $m_q(\varrho)$ traders, with the same effort that is required to deal with the m traders who are already in the market. We can think that he communicates, in any case, periodically with $m_q(\varrho) - m$ traders who are not in the market, and can thus increase the number of traders to $m_q(\varrho)$ with no additional effort. Higher effort is required only when increasing the number of traders beyond $m_q(\varrho)$.

was known to him, i.e., $\bar{q}^{-1}$, through various signalling schemes. With no loss in generality, let $\underline{e} = 0$.[10]

In addition to his role as a broker, broker A may trade the risky security on his own account. In contrast to traders, he realizes that his trades have an impact on the market price. We assume that broker A cannot trade in the period when the customer is trading, and he cannot engage in short-term trading, that is, in taking reverse positions during the same day or on two consecutive days.[11]

Broker A's objective at time $t = dT + 1$ is to maximize his expected utility, conditional on Ω_t, by setting his commission for the day, C_A^d, choosing his effort, e_t, the number of traders to contact, $m_t \in \{m, m_q(e_t)\}$, and his demand schedule for the risky asset, $y_t(P : P = P_t)$, subject to the two constraints regarding his trading that were described in the previous paragraph. At time $t = dT + 1$, he maximizes

$$\max_{C_A^d, e_t, m_t, y_t(P)} \mathrm{E}_t - \exp\left(-a\left[\sum_{s=1}^{2T}\left(y_s(v - P_s) - \frac{e_s^2}{2}\right) + \sum_d (1 - \pi_d)C_A^d\right]\right)$$

subject to :

(1) $sign(y_s) = sign(y_{s'})$ $\forall s$ and s' such that $y_s \neq 0$ and $y_{s'} \neq 0$

(2) $y_{dT+1} = 0$ $\forall d$.   (3.6)

Recall that π_d was defined as the probability that $b^d = D$. The first constraint states that once long always long and vice versa, whereas the second restricts the broker from trading in periods $dT + 1$. At time $t \neq dT + 1$, the maximization problem is similar, except that C_A^d is not chosen.

Timing: The timing of events is as follows. Periods $dT + 1$: First, the brokers set their commissions, $\{C_b^d\}_{b=A,D}$. After observing the brokers' commissions and her random daily endowment, x^d, the customer selects a broker, and submits to him a block order to sell (buy if $x^d < 0$) $\bar{x}^d$ units of the risky asset at the market price, while paying his commission. The selected broker receives the order and submits it to the exchange. Brokers select the level of their effort. Depending on broker A's effort and ability, either m or $m_q(e_t)$ traders arrive in the market and, along with broker A, indicate their demand schedules for the risky asset. The exchange selects a market price and trading occurs at the market price P_{dT+1}. In other periods $t \neq dT + 1$, there is no customer trading. Depending on broker A's action and ability, either m or $m_q(e_t)$ traders arrive in the market and, along with broker A, indicate their demand

---

[10] Let e_t^* denote broker A's effort in equilibrium. Given a normal distribution for his ability, there is a positive probability that $q - e_t^* < 0$ or $q - e_t^* > 1/m$. The latter would mean that broker A reduces the number of traders in the market. It is not clear what a negative $q - e_t^*$ would mean. As is commonly done in the literature, we ignore these possibilities by noting that the probability of these events can be made arbitrarily small by appropriate choices of $\bar{q}$ and θ^2 (as will be apparent later).

[11] We will discuss the role of these two assumptions in Section 3.5. The first assumption guarantees, among other things, that there is no conflict of interest between the broker and his customer in day one. We can therefore justify this assumption as a simple protection from lawsuits that might accuse the broker of breaching his fiduciary duty. The latter assumption, on the other hand, is purely a simplifying assumption that, in fact, reduces broker A's incentive to manipulate the closing price, by restricting his ability to take short-term positions.

schedules for the risky asset. Trading occurs at the market price P_t. The closing prices of the two days are P_{dT+T}.[12]

Equilibrium: An equilibrium consists of strategies for all agents that simultaneously maximize their objective functions, described in Eq. (3.3)–(3.6), taking as given other agents' strategies.

Denote by x_t the amount of assets sold by the customer in period t. Here $x_t = x^d$ when $t = dT + 1$ and $x_t = 0$ otherwise. The market price, $P_t(y_t, m_t, z_t, x_t)$, satisfies the market clearing condition when

$$y_t(P_t) + m_t(q, e_t) \cdot z_t(P_t) = x_t. \tag{3.7}$$

Before proceeding, we introduce some additional notation: Let $v_t = E_t v$ be the conditional expectation of the liquidation value, conditional on Ω_t, and $\sigma_t^2 = E_t[v - v_t]^2$, its conditional variance. It is straightforward to see that

$$\sigma_t^2 = [2T - t]\sigma_\varepsilon^2. \tag{3.8}$$

3.2. Equilibrium

In this section, we characterize the equilibrium of the game defined above. As there are only finitely many traders, and they are risk averse, both broker A's and the customer's trades have an impact on the price of the risky security.

The customer wishes to minimize the price impact of her trades. Broker A can help her to do so by attracting more traders to the market at the time her block is being traded. The increase in the number of traders reduces the price impact, as each trader then bears less risk. In equilibrium, broker A sets his commission low enough, as compared to broker D, to attract the customer's trades.

How large a commission broker A can charge on day d, C_A^d, depends on his reputation: i.e., on the customer's estimate of his ability q^{-1}. On day one, the customer revises her estimate of broker A's ability based on his performance on day zero. Her revised estimate is based on her day zero execution price, P_1, and the closing price P_T. As broker A wishes to influence this estimate, he exerts effort when executing her order in day zero, and manipulates the closing price, P_T, at the end of that day.

Proposition 1. *There exists an equilibrium in which the customer uses broker A on both days. Broker A searches out additional traders in the market and exerts an effort $e_1^* > \underline{e}$ in period 1. His effort is increasing in $\bar{x}^0, \bar{x}^1, a, \sigma_\varepsilon^2$ and θ^2. Broker A manipulates the day zero closing price of the risky asset, by buying a quantity y_T^* of shares (selling, if $y_T^* < 0$) from the m traders that come into the market in period T. Here y_T^* is negative (positive) if x^0 is positive (negative). The amount of manipulation $|y_T^*|$ is increasing in $\bar{x}^1, a, \sigma_\varepsilon^2$ and θ^2 and decreasing in m. It may increase or decrease in $\bar{x}^0$. The period 1 market price of*

---

the risky asset is

$$P_1 = v_1 - \frac{x^0 a \sigma_1^2}{m_1} = v_1 - x^0(q - e_1^*)a\sigma_1^2.$$

The day zero closing price is

$$P_T = v_T + \frac{y_T^* a \sigma_T^2}{m}$$

and in periods t, where $t \notin \{dT + 1, T\}$, $P_t = v_t$.

In period 1 there is a price impact from the customer's trade with P_1 being smaller (larger) than v_1, in the case where the customer is selling (buying). The price impact is a result of traders' risk aversion and it decreases with broker A's ability and effort, as these increase the number of traders in the market, m_1. Similarly, at close, broker A's manipulation affects the closing price. The closing price P_T is lower (higher) than the fair value of the asset, v_T, when broker A is selling (buying). Note that, in order to maximize the price impact of his trade, when manipulating, the broker does not search out additional traders in the market, but trades only with the m traders who come into the market in period T. Given the endogenous liquidity in the market, as m_t depends on his actions, broker A's trades have a large impact on the closing price. In periods where neither the customer nor the broker is trading, the price equals the asset's expected value.

Broker A manipulates and exerts effort in equilibrium, although it is costly for him, because through these actions he hopes to alter the customer's inference about his ability, to get a higher commission in day one. In equilibrium, the customer understands that the broker exerts effort and manipulates, and is able to make the right inference as to the broker's ability. Nevertheless, the broker is forced to act in this way, as otherwise the customer would infer that he had less ability.[13]

3.3. Theoretical implications of the model

The implications of the model are discussed below: First, Proposition 1 implies that both effort, e_1^*, that broker A exerts when executing the customer's day zero order, and the amount of day zero manipulation, $|y_T^*|$, increase in the day one order size, $\bar{x}^1$. This result follows, because the day one commission increases in $\bar{x}^1$, as is shown in the appendix.

Second, the effort and manipulation increase in the variance of broker A's ability, θ^2. The more uncertainty there is over the broker's ability, the more sensitive is the

---

[13] The basic idea about the manipulation of the learning process is similar to that in Holmstrom (1982). Fudenberg and Tirole (1986) also apply the concept and refer to it as "signal jamming." Interestingly, since the customer is able to make the right inference about the broker's ability, the equilibrium commission is the same as it would be in a setting where the broker could not manipulate. Therefore the customer does not lose from the manipulation, only the broker (the manipulator himself does). This might explain why, in the case of the Paris Bourse, the initiative to alter the closing price mechanism came from the institutional market participants themselves, not from the exchange.

day one commission to his perceived performance, and thus the increased incentive to manipulate and provide an effort.

Third, the effort and manipulation increase in the variance of the periodic shocks, σ_ε^2. This result is somewhat counter-intuitive, given that the periodic shocks create noise in observing the broker's execution quality. This occurs, however, because with high variance of the periodic shocks, there is high need for broker ability, and therefore broker A's commission becomes highly sensitive to the customer's estimate of his ability. For the same reason, effort and manipulation increase in the level of customer's risk aversion, a.

Fourth, the amount of manipulation is inversely related to m, the number of traders arriving periodically to the market. The intuition is simply that with a large number of traders at close, large m, manipulation is difficult and thus more costly.

Fifth, the day zero volume of customer trading, $\bar{x}^0$, affects the effort and manipulation. Higher $\bar{x}^0$ increases the price impact of the customer's trade in period 1 and makes the estimation of both the price impact and ability, q^{-1}, easier. This increases broker A's incentives to influence the price impact by exerting effort: thus e_1^* increases in $\bar{x}^0$. In contrast, the effect of $\bar{x}^0$ on manipulation is ambiguous. For a small $\bar{x}^0$ there is a small price impact, little updating on ability, and thus little reason to manipulate the closing price. As $\bar{x}^0$ increases, manipulation initially increases. However, when $\bar{x}^0$ is large, the customer can estimate the broker's ability accurately using only her transaction price, and therefore the reason to manipulate the closing price decreases.

Based on these results, we should expect to see most manipulation of the closing prices in volatile and illiquid stocks (small m), by brokers with an unknown ability (to the customer). In addition, manipulation is likely to follow large, but not too large, customer trades and precede large customer trades. As is shown in the appendix, the broker effort, in contrast to manipulation, is reflected in the day zero commission.

3.4. Empirical implications of the model

Our model makes several predictions on the price and volume behavior at close that could explain the obscure patterns at close in the Paris Bourse, described in Section 2. Direct calculation gives

$$var[P_T - P_{T-1}] - var[P_{T-1} - P_{T-2}]$$

$$= E[P_T - P_{T-1}]^2 - E[P_{T-1} - P_{T-2}]^2 = \left[\frac{y_T^* a \sigma_T^2}{m}\right]^2 > 0. \tag{3.9}$$

As Eq. (3.9) shows, there is an increase, proportional to y_T^{*2}, in price variability in the last period of day zero, due to price manipulation. Also, trading volume increases by $|y_T^*|$ in that period. Given Proposition 1, these patterns should be most pronounced on typically illiquid and volatile stocks. In addition, they should precede large customer trades the next day.

Our model also suggests a reason for the increase in hidden orders at close at the Paris Bourse. It could be that the hidden orders are other traders' attempts to encourage and take advantage of price manipulation at close. Making the order quantity hidden makes manipulation of prices seem easier. Our model is also consistent with the fact that the bursts in trading activity in the last minute of trading occur despite the increased bid–ask spread. It also offers an explanation as to why, during the last few seconds of the day, an increased proportion of orders is large and aggressive (see footnote 1).[14]

3.5. Model discussion and extensions

The model is based on several assumptions, discussed below.

Two periods: We assume two days, whereas, in reality, a broker and his customers interact repeatedly. In a similar model with several days, and constant broker ability, the uncertainty about a broker's type, and thus his reason to manipulate, would disappear over time. This suggests that we should see manipulation mainly after trades by new customers. Note, however, that in reality, due to employment changes, changes in market conditions, etc., the ability of a broker is unlikely to stay constant over time, and therefore brokers may need to prove themselves even to their long-standing customers.

Single active broker and customer: The main results of the paper would remain valid even if we assumed two (or more) active brokers. The broker with the best reputation would set his commission low enough to win the customer's day zero order. Both brokers' day one commissions would now depend on his performance. When expecting to win the customer also in day one, to influence her perception of his ability, the day zero broker would still try (a) to reduce the price impact through his effort and (b) reduce the customer's estimate of the price impact by closing price manipulation. However, in contrast to the basic model, if the day zero broker's performance was poor, the second broker might engage in manipulation to make the day zero broker's performance look worse, in order to win the customer's day one order and to increase his commission (this, we were told, is what sometimes happens).

If we had several customers using different brokers, many different situations could arise. Some brokers might have an incentive to manipulate the closing price downwards, others to manipulate it upwards. All we could then expect is high volatility and volume at close. Note that even in this case, there should be some manipulation in equilibrium, because if no one manipulates, then any given broker would have an incentive to do so.

Competitive traders: If the traders also internalize the effect of their trading on the market price, the distortion in prices due to manipulation would only increase. First,

---

[14] It can be shown, if we assume $\bar{x}^1 = \bar{x}^0$, that $y^* < \bar{x}^0$ and thus volume is lower at close than in period 1. This seems to be in contrast with the empirically observed fact that volume is highest around close. However, if we make the time of customer arrival uncertain, ex ante, we can have higher expected volume at close than at other points in time during the day.

broker A's incentive to manipulate would increase, as the sensitivity of his commission to the customer's estimate of his ability would increase. This occurs, as in this case, attracting new traders into the market would reduce the price impact not only through increased risk sharing, but also through increased competition. Second, the imperfect competition among traders would make manipulation of closing prices easier.

Broker A not allowed to trade when the customer trades: Instead of this assumption, we could have assumed that broker A can commit not to trade with his customer. In equilibrium, he would do so, whenever his day one commission would exceed the profits that he could make as a trader. It is crucial that broker A can somehow commit not to trade with his customer on day one. Otherwise, after receiving his commission, he would abandon his role as a broker, and become a trader instead. This commitment problem is a common incentive problem of brokers when trading directly with their customers: on the one hand, it is in their customer's interest that they minimize the price impact of trade by seeking additional traders to the market, but on the other hand, doing so reduces their own trading profits.[15]

Broker A not allowed to trade short term: This assumption is not crucial. If allowed to trade short term, e.g., in the case of customer selling, the broker would initially buy the asset from traders during periods $t \in \{2, T-1\}$ of day zero. At close, he would sell his acquired stock positions to manipulate the closing price downwards. Doing so would reduce broker A's cost from manipulation, as the time that he holds positions in the risky asset would be reduced.

Customer does not observe intraday security prices: First note that in the US, where intraday data has been available for quite some time (at a cost), the companies that evaluate brokers' execution quality, such as Elkins/McSherry, have often relied on a few key prices, such as open, high, low and close, when doing such evaluations (McSherry and Sofianos, 1998). If investors also use high and low to evaluate brokers, the same, although reduced, incentives to manipulate the closing price would exist. Note that, in this case, brokers would have an incentive also to manipulate the intraday high and low, not just the closing price. If a customer had the entire intraday data available to her, the broker might manipulate prices shortly before and after the customer's transaction.

Broker A does not perfectly know his own ability: We assume that broker A does not perfectly know his own ability and that he has communicated the part of his ability that is known to him, $\bar{q}^{-1}$, to the customer. These assumptions allow us to avoid any possible signalling of ability through commissions. What is crucial is that the customer (not necessarily the broker) is learning something from the broker's perceived performance. The results would look similar if we had signalling through commissions, in equilibrium, but the signal on the broker's ability was imperfect.

Symmetry: Finally, the model assumes a symmetric setting: e.g., execution quality is similar for both buy and sell orders. If we change this assumption, the

---

[15] Because of this, in different countries, various laws have been passed with an aim to guarantee the customers the best execution, by restricting brokers' ability to trade with their customers. It is not crucial whether broker A can trade with the customer on day zero.

manipulation, and the deviation from expected value at close, will vary depending on the type of the customer's order.

4. Optimal closing price mechanism

How should an exchange design its closing price mechanism, given the brokers' incentives to manipulate?

4.1. Theory on call auctions

Until now, we have assumed that the closing price is simply the last transaction price. In this section, we endogenize the closing price mechanism, and let the exchange design it, by allowing also for the possibility of a call auction.

The two criteria that the exchange is likely to be interested in minimizing are (1) trading interruptions and (2) the difference between the fair value of the asset and price at close. If the exchange wants to keep the market open over the same period of time, it might be able to improve the second objective merely by increasing the number of periods that are used to calculate the closing price. This approach was initially adopted by the stock exchanges of London, Milan and Madrid, where the closing price was calculated as a weighted average of prices near the close. A second alternative, recently introduced by the Paris Bourse, and later adopted also by London, Milan and Madrid, among others, is to have a call auction at close. We now consider whether a call auction can reduce volatility at close on day zero. When setting up a call auction at the end of the day, the exchange interrupts trading for $K \geqslant 1$ periods, $\{T - K, \dots, T - 1\}$, to allow orders to accumulate. After this, a call auction is performed in period T. Let us assume that all traders who would have arrived in the market during the K periods, when the market is closed, arrive in the call auction, implying that $(K + 1)m$ traders are present in the call auction. The previously assumed closing price mechanism, where closing price is the last transaction price, corresponds to the case where $K = 0$.

Proposition 2. *Assume a $K < T - 2$ period trading interruption and a call auction at close. There exists an equilibrium in which the customer uses broker A on both days. Broker A searches out additional traders in the market and exerts a constant effort $e_1^* > 0$ in period 1. He manipulates the closing price of the risky asset at the end of day zero, by buying a constant quantity*

$$y_T^{**} = \left(\frac{m + 2}{(K + 1)m + 2} \right) y_T^*$$

of shares (selling, if $y_T^{**} < 0$) from the $(K + 1)m$ traders that come into the market in period T. In period 1 the price is

$$P_1 = v_1 - x^0(q - e_1^*)a\sigma_1^2.$$

The closing price for day zero is

$$P_T = v_T + \frac{y_T^{**} a \sigma_T^2}{(K+1)m} \qquad (4.1)$$

and in periods t, where $t \notin \{dT+1, T\}$, $P_t = v_t$.

Proposition 2 shows that broker effort and the price impact of the customer's trade remain the same as they were in the absence of a call auction. However, there is now less manipulation ($y_T^{**} < y_T^*$) and, because of this, and the increased number of traders at close, the closing price is nearer to the expected value of the risky asset, v_T. The reason for less manipulation is simply that with $(K+1)m$ traders in the market at close, prices are more difficult to manipulate, than with m traders in the market, and therefore broker A manipulates less, in equilibrium.

The next issue is the determination of the optimal closing auction. We assume that the exchange minimizes the following loss function L:

$$\min_{K \geqslant 0} L = \min_{K \geqslant 0} (\omega_k K + \omega_{\sigma^2} var[P_T - v_T]), \qquad (4.2)$$

when determining the optimal length of trading interruption, prior to a call auction. Here, ω_k and ω_{σ^2} are the weights that the exchange attaches to a trading interruption and excessive price variability, respectively. For simplicity, we have assumed a linear objective function for the exchange.

In the case of a call auction at close, direct calculation shows that the excessive price variability of the closing price in day zero is

$$var[P_T - v_T] = \left[\frac{(m+2)y_T^* a \sigma_T^2}{(K+1)m((K+1)m+2)} \right]^2. \qquad (4.3)$$

Note that this is lower in the case of a call market ($K > 0$) than in the absence of one ($K = 0$). Substituting this to (4.2), and letting K^* denote the optimal period of trading interruption prior to a call auction, we obtain the following result.

Proposition 3. *The optimal closing call auction has the following characteristics: There exists $\underline{\omega}_k$, such that when $\omega_k \geqslant \underline{\omega}_k$, $K^* = 0$. When $\omega_k < \underline{\omega}_k$, $K^* > 0$. K^* is decreasing in m and ω_k, but increasing in ω_{σ^2}, σ_ε^2 and θ^2.*

The proposition states that exchanges which value continuous trading highly (ω_k is large) should have their closing price equal the price of the last trade. Others should adopt a call auction at close. When designing the call auction, the optimal period of temporary market closure, K^*, should be smaller, the larger the liquidity, m, and larger, the more volatile the stock, σ_ε^2. These results are intuitive as the level of manipulation increases in σ_ε^2, but decreases in m.

Furthermore, as K^* (and the level of manipulation) depends on θ^2, the degree of uncertainty about the broker's ability, the period of market closure could also depend on some observable measure of the stability of the investor base, such as the

proportion of foreign ownership. In the Paris Bourse, the period of market closure is always the same.[16]

4.2. Other closing price mechanisms

It can be shown also that the second alternative to calculate the closing price, which we have discussed, i.e., calculating the closing price as a weighted average of prices near the close, would reduce manipulation and excessive price variability at close. This method avoids any interruption in trading. However, by also using prices before the close, it does not fully take into account the changes in the value of the asset that occur just before the close.

A third alternative, in use at the NYSE, is to have uninterrupted trading, but to allow "orders at close," i.e., orders that are executed at the closing price. When there is significant use of such contracts, this system can reduce manipulation without a trading interruption. One benefit of the closing auction is that, by restricting trading opportunities, it forces traders to trade at close. In addition to those mentioned, some exchanges have adopted other closing price mechanisms. For instance, in Hong Kong the closing price is calculated as an average of a random sample of prices near the close, whereas in London SEAQ, the closing price is simply the mid price of the best bid and ask of the market makers. In New Zealand, the closing price equals the price of the last trade, but the exchange closes trading, on any given stock, at a random point in time. Table 1 provides information on closing price mechanisms in different exchanges.

4.3. Empirical evidence from Paris and Madrid

How has the introduction of a call auction at close altered the observed patterns at close? There is evidence that after the implementation of the call auction, the abnormal price volatility for an average stock decreased by 25%. There was no longer abnormal volatility and the returns were no longer significantly different from zero (Thomas, 1998). This is consistent with our model that predicts that volatility decreases as an exchange introduces a call auction at close. We take these observations as evidence that the call auction, as our model suggests, has succeeded in reducing price manipulation at close.

One additional piece of evidence on closing price manipulation, and the effect of a change in the closing price mechanism, comes from Spain. Starting March 26, 1998, the closing price at the Bolsa de Madrid was calculated as the value-weighted average price of the last 500 shares traded. Moreover, if the absolute price change in the last minute of trading exceeded a certain range, chosen by the exchange, the

---

[16] It follows from the first order condition to (4.2) that K^* increases also in the expected day one trading volume, $\bar{x}^1$ (as does the level of manipulation). However, as is the case for manipulation, the results with respect to day zero customer trading volume, $\bar{x}^0$, are ambiguous: The optimal length of the market closure initially increases but later decreases in $\bar{x}^0$.

Table 1
Closing price determination in selected exchanges (May 2003)

Country/exchange	Closing price mechanism
Australia	Call auction
Brazil	Call auction
Canada (Toronto)	Last traded price
China (Shanghai)	Volume weighted average of prices in last minute of trading
Denmark	Last traded price
Finland	Last traded price
France	Call auction
Germany (XETRA)	Call auction
Hong Kong	Median price of five observed prices in last minute of trading
Italy	Call auction
Japan (Tokyo)	Last traded price. Market participants can place orders at close
Korea	Call auction
New Zealand	Last traded price. Random closing time between 3.55–4.00 pm
Spain	Call auction
Sweden	Call auction
Switzerland	Call auction
Turkey	Last traded price
UK (SETS)	Call auction
UK (SEAQ)	Best bid and ask of the market makers
USA (Amex)	Last traded price
USA (Nasdaq)	Last traded price
USA (NYSE)	Last traded price. Market participants can place orders at close

Source: various stock exchanges.

closing price was computed as a value-weighted average of the prices observed in the last five minutes of trading. To study the effect of this change in the closing price mechanism on the trading behavior near close, we studied a sample of all transactions during the last five minutes of trading in the two-month time interval surrounding the change in the closing price mechanism. One sign that the new closing price mechanism hampered manipulation was that, from the month preceding the change to the month following the change, the number of transactions in the last 15 seconds of the day decreased by 15%.[17,18]

One sign of continued closing price manipulation at the Madrid Stock Exchange after the introduction of this closing mechanism was, however, that after the change, the number of transactions with exactly 500 shares traded increased

---

[17] A transaction where one broker sells, say, a thousand stocks to two other brokers, 500 each, is counted as two transactions. On some days, there were transactions which were stamped a few seconds after the market close. We included these transactions in our calculations.

[18] Despite the decrease in the total number of transactions, following the change in the closing price mechanism, the volume in the last 15 seconds of the day actually increased by 40%. Although the variance in trading volume is too large to allow good statistical inference, it could be that reduced closing price manipulation encourages others to trade larger volumes at close.
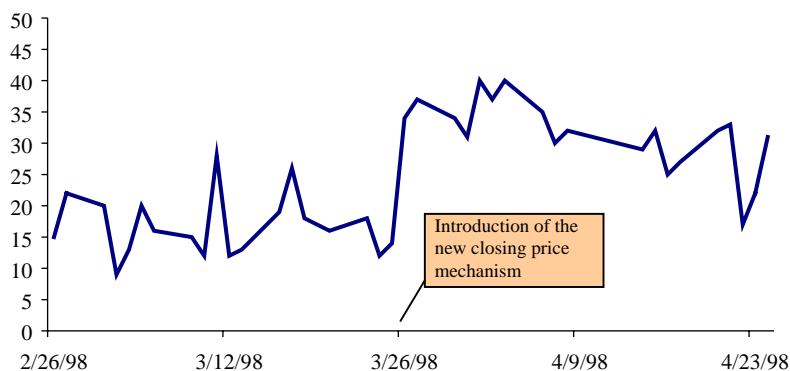
Fig. 4. Number of transactions with exactly 500 shares traded in the last 15 seconds of the day at the Bolsa de Madrid.

dramatically during the last minute, and especially during the last 15 seconds of the day (Fig. 4).[19]

In June 2000, the Madrid Stock Exchange revised again its closing price mechanism by adopting a closing call auction with a five-minute trading interruption prior to the auction. According to Rodriguez (2000), and consistent with the evidence from Paris, the introduction of the call auction resulted in a significant decrease in the five-minute volatility prior to close.

5. Conclusion

We have presented an agency-based model of price manipulation in which a broker manipulates the closing price of a stock in order to give a better impression of his execution quality to his customer. The contribution of this paper is to show that manipulation can occur due to agency reasons. Our model is consistent with many of the patterns observed during the last minute of trading at the Paris Bourse. We have shown, theoretically, that introducing a closing call auction, as was done at the Paris Bourse, reduces manipulation and brings the closing price nearer to the fair value of the asset.

The increased availability of intraday data may reduce brokers' incentives to manipulate prices, if it leads to increased adoption of other reference prices to evaluate broker performance that are more difficult to manipulate than the closing price, such as the Volume Weighted Average Price. Note, however, that even a free

---

[19] In fact, there were 318 such transactions within the last 15 seconds of the day one month before the implementation of this rule, whereas the corresponding figure after was 598. At this level, the transactions for exactly 500 shares, during the last 15 seconds, accounted for over 12% of all transactions during the last 15 seconds and for nearly 7% of all transactions during the last minute of trading. Before the change in the closing price mechanism, the corresponding figures were 5.6% and 3.5%, respectively.

availability of intraday data need not entirely remove the incentive to manipulate the learning process of the customer by price manipulation. Ideally, with access to intraday data, a customer should compare her transaction price with other transaction prices close to the time period of her transaction. In this case, however, the broker might manipulate the prices near the time of the customer's transaction.

Finally, there are many other similar situations in which a financial institution may have an agency-based incentive to manipulate securities prices, besides the one that we have described. For instance, an investment bank, prior to executing a M&A transaction, may be interested in manipulating the prices of the stocks of the companies in question. The incentive for manipulation in this case could come from both a value-based commission and the reputational gain from managing high value transactions (in this case, manipulation could also take the form of passing information to the press). When the acquisition price is based on an average of past closing prices, or one firm's stock is being used as a medium of payment, an additional incentive to manipulate may be present. These are just a few examples of the potentially many reasons for agency-based manipulation that future research can explore.

Appendix A

Proof of Proposition 1. We start by analyzing how the customer's trading results in a price impact of trade.

Using our notation, it can be shown that the demand of each trader in period t is

$$z_t(P_t) = \frac{v_t - P_t}{a\sigma_t^2}. \tag{A.1}$$

Market clearing implies that

$$y_t + m_t \left[\frac{v_t - P_t}{a\sigma_t^2} \right] = x_t, \tag{A.2}$$

which implies

$$P_t = v_t + \frac{y_t a\sigma_t^2}{m_t} - \frac{x_t a\sigma_t^2}{m_t}. \tag{A.3}$$

Here, m_t is either m or $m_q = (q - e_t)^{-1}$, depending on broker A's action.

The following lemma characterizes the customer's posterior beliefs on the ability of broker A, after observing P_1 and P_T and assuming agents behave according to Proposition 1.

Lemma 1. *The posterior distribution $f(q|\Psi_{T+1})$ is normal with mean*

$$\bar{q}_{T+1} = c_0 - c_1 P_1 + c_2 P_T \tag{A.4}$$

and a constant variance θ_{T+1}^2. c_1 and c_2, are positive (negative) constants in the case where the customer is selling (buying).

Proof. The customer essentially observes two independent noisy signals on v_1,

$$z_1 = v_T = P_T - \frac{y_T^* a\sigma_T^2}{m} = v_1 + \sum_2^T \varepsilon_t \tag{A.5}$$

and

$$z_2 = P_1 + x^0 a\sigma_1^2(\bar{q} - e_1^*) = v_1 - x^0 a\sigma_1^2(q - \bar{q}). \tag{A.6}$$

His prior on v_1 is also normally distributed with mean $\bar{v}$ and variance σ_ε^2. z_1, z_2, and v_1 are jointly normally distributed: $z_1 \sim N(v_1, \sigma_{T+1}^2)$ and $z_2 \sim N(v_1, (x^0 a\sigma_1^2)^2 \theta^2)$. Using a well-known result for normal distributions, the posterior distribution for v_1, after the two signals, $f(v_1|z_1, z_2)$ is normal with mean

$$E[v_1|z_1, z_2] = \frac{\dfrac{\bar{v}}{\sigma_\varepsilon^2} + \dfrac{z_1}{\sigma_{T+1}^2} + \dfrac{z_2}{(x^0 a\sigma_1^2\theta)^2}}{\dfrac{1}{\sigma_\varepsilon^2} + \dfrac{1}{\sigma_{T+1}^2} + \dfrac{1}{(x^0 a\sigma_1^2\theta)^2}} = \frac{\dfrac{\bar{v}}{\sigma_\varepsilon^2} + \dfrac{P_T - \dfrac{y_T^* a\sigma_T^2}{m}}{\sigma_{T+1}^2} + \dfrac{P_1 + x^0 a\sigma_1^2(\bar{q} - e_1^*)}{(x^0 a\sigma_1^2\theta)^2}}{\dfrac{1}{\sigma_\varepsilon^2} + \dfrac{1}{\sigma_{T+1}^2} + \dfrac{1}{(x^0 a\sigma_1^2\theta)^2}} \tag{A.7}$$

and variance

$$var[v_1|z_1, z_2] = \frac{1}{\dfrac{1}{\sigma_\varepsilon^2} + \dfrac{1}{\sigma_{T+1}^2} + \dfrac{1}{(x^0 a\sigma_1^2\theta)^2}}. \tag{A.8}$$

The posterior distribution $f(q|\Psi_{T+1})$ is then also normal with mean

$$E[q|\Psi_{T+1}] = \bar{q}_{T+1} = \frac{E[v_1|z_1, z_2] - P_1}{x^0 a\sigma_1^2} + e_1^* = c_0 - c_1 P_1 + c_2 P_T, \tag{A.9}$$

where

$$c_0 = c_2 \left[(T-1)\bar{v} + \frac{\sigma_{T+1}^2(\bar{q} - e_1^*)}{x^0 a\sigma_1^2\theta^2} - \frac{y_T^* a\sigma_T^2}{m} \right] + e_1^*,$$

$$c_1 = \frac{x^0 a\sigma_1^2\theta^2}{(x^0 a\sigma_1^2\theta)^2 + \dfrac{\sigma_{T+1}^2}{T}},$$

$$c_2 = \frac{c_1}{T}$$

and a variance

$$var[q|\Psi_{T+1}] = \theta_{T+1}^2 = \left[\frac{var[v_1|z_1, z_2]}{(x^0 a\sigma_1^2)^2} \right] = \frac{1}{\dfrac{T(x^0 a\sigma_1^2)^2}{\sigma_{T+1}^2} + \dfrac{1}{\theta^2}}. \qquad \square \tag{A.10}$$

Day one: Given that the day one commission is paid independently of broker A's (unobservable) effort, broker A exerts no effort on day one: $e_t = 0$, $\forall t > T$.

Similarly, it is easy to see that $y_t = 0$, $\forall t > T$. In equilibrium, broker A sets his day one commission, C_A^1, just low enough to obtain the customer's order, in case $C_D^1 = 0$, and broker D sets $C_D^1 = 0$. Otherwise, one of the two would be better off changing his commission, given the other broker's strategy. With $C_D^1 = 0$, the customer's expected utility from using broker D at time $T + 1$ is

$$U_{C,T+1}^D = \mathrm{E}\left(-\exp\left(-ax^1\left[v_{T+1} - \frac{x^1 a \sigma_{T+1}^2}{m}\right] - ax^0 P_1 + aC_{b^0}^0\right)|\Psi_{T+1}\right). \quad (A.11)$$

Similarly, when broker A's day one commission is set at C_A^1, the customer's expected utility from using broker A, who searches out $m_q(\varrho)$ traders, is

$$U_{C,T+1}^A = \mathrm{E}(-\exp(-ax^1[v_{T+1} - qx^1 a \sigma_{T+1}^2] + aC_A^1 - ax^0 P_1 + aC_{b^0}^0)|\Psi_{T+1})$$

$$= U_{C,T+1}^D \cdot \exp\left((a\bar{x}^1 \sigma_{T+1})^2\left[\bar{q}_{T+1} - \frac{1}{m} + \frac{(a\bar{x}^1 \sigma_{T+1} \theta_{T+1})^2}{2}\right] + aC_A^1\right)$$

$$\quad (A.12)$$

given that by Lemma 1: $f(q|\Psi_{T+1}) \sim \mathrm{N}(\bar{q}_{T+1}, \theta_{T+1}^2)$.

Broker A charges as high a commission as possible without losing the deal to broker D. He therefore maximizes his utility by setting C_A^1 so that $U_{C,T+1}^A = U_{C,T+1}^D$, or by setting

$$C_A^1 = a(\bar{x}^1 \sigma_{T+1})^2\left[\frac{1}{m} - \bar{q}_{T+1} - \frac{(a\bar{x}^1 \sigma_{T+1} \theta_{T+1})^2}{2}\right]. \quad (A.13)$$

Here, we have assumed that $\bar{q}_{T+1} < 1/m$ and that θ_{T+1}^2 is small enough so that C_A^1 is strictly positive. As one would expect, the optimal commission is increasing with the estimated ability of broker A.

Day zero: We now show that the expectations of broker A's behavior, in terms of effort and trading, are rational, assuming that $b^0 = A$. Taking expectations of broker A's expected utility (3.6) at time T, substituting for $y_s = e_s = 0$, $\forall s > T$, $\pi_1 = 0$, C_A^1, using (A.13) and (A.9), and for the market price in period T,

$$P_T = v_T + \frac{y_T a \sigma_T^2}{m_T}, \quad (A.14)$$

broker A's maximization problem at time T can be written as

$$\max_{e_T, m_T, y_T} U_{A,T} = \max_{e_T, m_T, y_T} -\exp\left(-aC_A^0 - aC_A^1 - a\sum_{s<T} y_s(v_T - P_s) + \frac{y_T^2 a^2 \sigma_T^2}{m_T}\right)$$

$$\times \exp\left(\frac{(\sum_{s<T} y_s + y_T)^2 a^2 \sigma_T^2}{2} + \frac{a}{2}\sum_{t=1}^T e_t^2\right), \quad (A.15)$$

where

$$aC_A^1 = (a\bar{x}^1 \sigma_{T+1})^2\left[\frac{1}{m} - c_0 + c_1 P_1 - c_2\left(v_T + \frac{y_T a \sigma_T^2}{m_T}\right) - \frac{(a\bar{x}^1 \sigma_{T+1} \theta_{T+1})^2}{2}\right].$$

It is now easy to see, given that $a(\bar{x}^1 \sigma_{T+1})^2|c_2| > |y_T|$, in equilibrium, that (A.15) is maximized by setting $m_T = m$ and $e_T = 0$. It is also easy to show that $y_t = 0$ for all

$t < T$ and $e_t = 0 \ \forall t \neq 1$. Using this in (A.15) and taking the first order condition with respect to y_T, gives

$$y_T = y_T^* = \frac{-a(\bar{x}^1 \sigma_{T+1})^2 c_2}{m+2} = -\left[\frac{x^0(\bar{x}^1 a \sigma_1 \sigma_{T+1} \theta)^2}{(m+2)[T(\bar{x}^0 a \sigma_1^2)^2 \theta^2 + \sigma_{T+1}^2]}\right]. \tag{A.16}$$

Eq. (A.16) implies all the comparative statics results regarding manipulation stated in Proposition 1. Using the results that $y_t = 0$ for all $t \neq T$ and $e_t = 0 \ \forall t \neq 1$, taking expectations of (3.6) at time 1, substituting for C_A^1, using (A.13), (A.9) and (A.3), and taking the first order condition with respect to e_1, gives

$$e_1^* = x^0 (a \bar{x}^1 \sigma_{T+1} \sigma_1)^2 c_1 = \frac{a^3 (\bar{x}^0 \bar{x}^1 \sigma_{T+1} \sigma_1^2 \theta)^2}{(\bar{x}^0 a \sigma_1^2 \theta)^2 + \dfrac{\sigma_{T+1}^2}{T}}. \tag{A.17}$$

Eq. (A.17) implies all the comparative statics results regarding effort stated in Proposition 1.

We now look at broker A's commission in day zero. Solving for the maximum commission C_A^0 that broker A can charge without losing the customer (similarly as for C_A^1) gives

$$C_A^0 = a(\bar{x}^0 \sigma_1)^2 \left[\frac{1}{m} - (\bar{q} - e_1^*) - \frac{(a \bar{x}^0 \sigma_1 \theta)^2}{2}\right]. \tag{A.18}$$

Here we have assumed that θ is small enough so that C_A^0 is positive. As Eq. (A.18) shows, the broker effort, in contrast to manipulation, is reflected in the day zero commission.

When C_A^0 is positive, broker A's expected utility from setting this commission exceeds his utility from setting a higher commission in day zero, and letting broker D handle the customer's day zero order. Even though broker A's day one commission becomes uncertain if $b^0 = A$, the disutility from the increased uncertainty is compensated for by a higher expected commission in day one, as the customer's uncertainty over broker A's ability is reduced. Denoting by $U_{A,1}$ broker A's expected utility in period one under the equilibrium strategy, taking expectation of (3.6) in period one, using (A.13), we obtain the result that broker A's expected utility from setting a higher commission is

$$U_{A,1} \exp\left(a C_A^0 + \frac{(a \bar{x}^1 \sigma_{T+1})^4}{2}\left[\theta^2 - \theta_{T+1}^2\right] - \frac{(a \bar{x}^1 \sigma_{T+1})^4 var(\bar{q}_{T+1})}{2}\right)$$

$$= U_{A,1} \exp(a C_A^0) < U_{A,1}. \qquad \square$$

Proof of Proposition 2. The proof is similar to that of Proposition 1 and is therefore omitted. The only difference from the proof of Proposition 1, apart from a slightly different expression to c_0 in Lemma 1, is that, when maximizing Eq. (3.6), broker A cannot set m_T below $(K+1)m$, and therefore $m_T = (K+1)m$. The first-order conditions to (3.6) imply that $e_1 = e_1^*$ and $y_T = y_T^{**} = ((m+2)/((K+1)m+2))y_T^*$.

Proof of Proposition 3. We can easily verify that the objective function (4.2) is convex with respect to K. If $L(K = 0) \leqslant L(K = 1)$, the minimum is characterized by $K^* = 0$. This occurs when

$$\omega_k \geqslant \underline{\omega}_k \equiv \omega_{\sigma^2} \frac{[y_T^* a \sigma_T^2]^2}{m^2} \left[1 - \frac{(m+2)^2}{4(2m+2)^2} \right].$$

Otherwise, the minimum is characterized by $K^* > 0$. Because of convexity of the loss function L, with respect to K, the minimum is attained by selecting K^* equal to the smallest integer K such that $L(K) - L(K+1) < 0$, or

$$\omega_{\sigma^2} \Psi \left(\left[\frac{1}{(K+1)m((K+1)m+2)} \right]^2 - \left[\frac{1}{(K+2)m((K+2)m+2)} \right]^2 \right) - \omega_k < 0,$$

where

$$\Psi = \left[\frac{a^3 x^0 (\bar{x}^1 \sigma_1 \sigma_{T+1} \sigma_T \theta)^2}{[T(\bar{x}^0 a \sigma_1^2 \theta)^2 + \sigma_{T+1}^2]} \right]^2.$$

The comparative statics results follow as the left-hand side of this equation is monotonically decreasing in K, ω_k and m, and monotonically increasing in $\omega_{\sigma^2}, \theta^2$ and σ_ε^2 (remember that σ_t^2 is proportional to σ_ε^2). $\square$

References

Admati, A., Pfleiderer, P., 1988. A theory of intraday patterns: volume and price variability. Review of Financial Studies 1, 3–40.

Brock, W., Kleidon, A., 1992. Periodic market closure and trading volume. Journal of Economic Dynamics and Control 16, 451–489.

Cherian, J.A., Jarrow, R.A., 1995. Market manipulation, In: Jarrow, R.A., Maksimovic, V., Ziemba, W.T. (Eds.), Handbooks in Operations Research and Management Science, Vol. 9: Finance. North-Holland, Amsterdam, pp. 611–630.

Cushing, D., Madhavan, A., 2000. Stock returns and trading at the close. Journal of Financial Markets 3, 45–67.

Felixson, K., Pelli, A., 1999. Day end returns—stock price manipulation. Journal of Multinational Financial Management 9, 95–127.

Foster, D., Viswanathan, S., 1990. A theory of the interday variations in volume, variance, and trading costs in securities markets. Review of Financial Studies 3, 593–624.

Fudenberg, D., Tirole, J., 1986. A signal-jamming theory of predation. Rand Journal of Economics 17, 366–376.

Grossman, S., Miller, M., 1988. Liquidity and market structure. Journal of Finance 43, 617–637.

Harris, L., 1989. A day-end transaction price anomaly. Journal of Financial and Quantitative Analysis 24, 29–45.

Hong, H., Wang, J., 2000. Trading and returns under periodic market closures. Journal of Finance 55, 297–354.

Holmstrom, B., 1982. Managerial incentive problems: a dynamic perspective. Essays in Economics and Management in Honor of Lars Wahlbeck. Swedish School of Economics, Helsinki.

Kumar, P., Seppi, D.J., 1992. Futures manipulation with cash settlement. Journal of Finance 47, 1485–1502.

Madhavan, A., Cheng, M., 1997. Block trades in the upstairs and downstairs markets. Review of Financial Studies 10, 175–203.

McInish, T., Wood, R.A., 1990. A transaction data analysis of the variability of common stock returns during 1980–1984. Journal of Banking and Finance 14, 113–129.

McSherry, R., Sofianos, G., 1998. US institutional investor trading in NYSE-listed non-US stocks. Global Equity Markets Conference Proceedings, Bourse de Paris-NYSE.

Rodriguez, J.A., 2000. Fijacion del precio de cierre en el sistema de interconexion bursatil Espanol: cierre mediante cambios medios versus subasta de cierre. Mimeo, Sociedad de Bolsas, S.A.

Seppi, D., 1990. Equilibrium block trading and asymmetric information. Journal of Finance 45, 73–94.

Thomas, S., 1998. End of day patterns on the paris bourse after implementation of a call auction. Global Equity Markets Conference Proceedings, Bourse de Paris-NYSE.